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FUZZY CONTROL OF STRUCTURAL VIBRATION. AN ACTIVE MASS SYSTEM DRIVEN BY A FUZZY CONTROLLER

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SUMMARY

The authors are engaged in a long-term research project studying the potential of fuzzy control strategies for active structural control in civil engineering applications. The advantage of this approach is its inherent robustness and its ability to handle the non-linear behaviour of the structure. Moreover, the computations for driving the controller are quite simple and can easily be implemented into a fuzzy chip. In this paper attention is focused on the response of a three-storey frame, subjected to earthquake excitation, controlled by an active mass driver located on the top floor. The design and the implementation of the controller driving the AMD system are discussed. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: active mass driver; fuzzy control; fuzzy logic; structural control

1. INTRODUCTION

In the design of an active controller, the goal is the reduction of the structural response in term of accelerations, velocities and displacements under the limitation of both the control force level (limited by the actuators feature and by the required amount of energy) and the number of measured signals. Fuzzy theory has been recently[1–4] proposed for the active structural control of civil engineering systems. As an alternative to classical control theory, it allows the resolution of imprecise or uncertain informations[5]. Moreover fuzzy control can handle the hysteretic behaviour of structures under earthquake.[2,6]

The main advantages in adopting a fuzzy control schemes can be summarized as follows.

1. The uncertainties of input data from the ground motion and structural vibrations sensors are treated in a much easier way by fuzzy control theory than by classical control theory. Fuzzy logic, which is the basis of the fuzzy controller, intrinsically accounts for such uncertainties. The implementation of fuzzy controllers makes use of linguistic synthesis and therefore they are not affected by the selection of a specific mathematical model. As a consequence the resulting fuzzy controller possesses an inherent robustness.
2. The whole fuzzy controller can be easily implemented in a fuzzy chip, which guarantees immediate reaction times and autonomous power supply.[7]
3. The knowledge base identifies the actual variables driving the control process: in the specific benchmark problem developed throughout this paper only two variables must be measured and estimated to implement the controller.

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4. The benchmark is assuming the linear model is consistent with the real structural system. For a more realistic implementation, at least geometric non-linearities should be incorporated in the problem. The fuzzy controller does not require modifications to follow such a case.

2. FUZZY INFERENCE

Fuzzy control converts a linguistic control strategy into an automatic control strategy. The classic fuzzy inference scheme consists of the following steps:[2,4,6]

1. fuzzification interface (the controller input variables, measured from the structure, are fuzzified into linguistic terms),
2. knowledge base (consisting of fuzzy *IF–THEN* rules and membership functions),
3. fuzzy reasoning (resulting in a fuzzy output for each rule),
4. defuzzification interface (providing the crisp control signal).

In this paper, the preliminary design of the controller will couple the Larsen's min product rule, to combine the membership values for each rule, with the centre of gravity (COG) defuzzification scheme, to obtain the output crisp value. The controller can also be optimized by the algorithm that uses the Takagi and Sugeno[8] inference system. This computes the fuzzy output for each rule as a linear combination of input variable membership values plus a constant term. The final crisp output is achieved using a weighted average. Within this paper the authors did not pay attention to the optimization of the controller.

This is mainly due to three reasons:

1. The authors wish first to pursue a laboratory validation of the controller; this, in particular, drove the authors selection between the benchmark options.[9] The authors decided to study the Active Mass Driver System example, which can easily be scaled for matching the available testing facilities, rather than the full-scale tendon system implemented at the National Center of Earthquake Engineering Research in Buffalo.
2. Optimization can easily be pursued by using a neuro-fuzzy scheme,[2] but for this purpose a family of excitation time histories is required.
3. Adaptive fuzzy controllers are the final goal of the ongoing research effort toward the evaluation of fuzzy control potential.[10]

3. THE BENCHMARK PROBLEM

Consider a structural system subjected to an earthquake ground acceleration. The equations of motion in the state vector form are:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{E}\mathbf{w} \tag{1}$$

$$\dot{\mathbf{y}} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} + \mathbf{F}\mathbf{w} \tag{2}$$

In equation (1) and (2) $\mathbf{x}$ is the state vector, $\mathbf{y}$ the vector of the measured quantities, $\mathbf{u}$ the control force and $\mathbf{w}$ the external excitation.

One starts from the knowledge of the matrices **A**, **B**, **C**, **D**, **E** and **F** of the control problem.[9] Their dimension are 28×28, 28×1, 13×28, 13×1, 28×1, 13×1, respectively. This reduces vectors $\mathbf{u}$ and $\mathbf{w}$ to the scalars u and w. Moreover, the benchmark formulation distinguishes two blocks, in the matrices **C**, **D** and **F**, of six rows the first subset and of seven rows the second subset. No use of the latter block is done in this paper. The six components of the first block are $[x_m, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, \ddot{x}_{am}, \ddot{x}_g]$, namely, the Active Mass Driver displacement, the three storey absolute accelerations, the AMD absolute acceleration and the ground acceleration.

Analog-digital conversions, time delays and similar implementation aspects are all incorporated in the numerical tool of solution. Among them an estimation of the velocities $\dot{x}_{ai}$ from the measured accelerations $\ddot{x}_{ai}$ is available. Different selections for $w(t)$ are suggested: the classical El-Centro record, the signal recorded in Hachinohe as well as realizations of a stationary filtered Gaussian white-noise.

The following performance indexes are considered:

$$J_1 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{d_1}}{\sigma_{x_{3o}}}, \frac{\sigma_{d_2}}{\sigma_{x_{3o}}}, \frac{\sigma_{d_3}}{\sigma_{x_{3o}}} \right\}$$

$$J_2 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{\ddot{x}_{a1}}}{\sigma_{\ddot{x}_{a3o}}}, \frac{\sigma_{\ddot{x}_{a2}}}{\sigma_{\ddot{x}_{a3o}}}, \frac{\sigma_{\ddot{x}_{a3}}}{\sigma_{\ddot{x}_{a3o}}} \right\}$$

$$J_3 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{x_m}}{\sigma_{x_{3o}}} \right\}$$

$$J_4 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{\dot{x}_m}}{\sigma_{\dot{x}_{3o}}} \right\}$$

$$J_5 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{\ddot{x}_{am}}}{\sigma_{\ddot{x}_{a3o}}} \right\}$$

$$J_6 = \max_{\text{El-Centro, Hachinohe}} \left[\max_t \left\{ \frac{|d_1(t)|}{x_{3o}}, \frac{|d_2(t)|}{x_{3o}}, \frac{|d_3(t)|}{x_{3o}} \right\} \right]$$

$$J_7 = \max_{\text{El-Centro, Hachinohe}} \left[\max_t \left\{ \frac{|\ddot{x}_{a1}(t)|}{\ddot{x}_{a3o}}, \frac{|\ddot{x}_{a2}(t)|}{\ddot{x}_{a3o}}, \frac{|\ddot{x}_{a3}(t)|}{\ddot{x}_{a3o}} \right\} \right]$$

$$J_8 = \max_{\text{El-Centro, Hachinohe}} \left[\max_t \frac{|x_m(t)|}{x_{3o}} \right]$$

$$J_9 = \max_{\text{El-Centro, Hachinohe}} \left[\max_t \frac{|\dot{x}_m(t)|}{\dot{x}_{3o}} \right]$$

$$J_{10} = \max_{\text{El-Centro, Hachinohe}} \left[\max_t \frac{|\ddot{x}_{am}(t)|}{\ddot{x}_{a3o}} \right]$$

where σ_{d_i} is the root mean square (rms) interstorey drift for the ith floor, $\sigma_{\ddot{x}_{ai}}$ is the rms of the ith floor acceleration, σ_{x_m}, $\sigma_{\dot{x}_m}$, $\sigma_{\ddot{x}_{am}}$ are the rms of the relative displacement and velocity and of the absolute acceleration of the AMD mass, $\dot{x}_i$ and $\ddot{x}_{ai}$ are the relative velocity and the absolute acceleration of the ith floor, respectively. The normalization values in $\zeta_1, \ldots, \zeta_5$ are: $\sigma_{x_{3o}} = 1{\cdot}31$ cm, $\sigma_{\dot{x}_{3o}} = 47{\cdot}9$ cm/s, $\sigma_{\ddot{x}_{a3o}} = 1{\cdot}79$ cm/s^2. ω_g, ζ_g are the Kanai–Tajimi parameters: according to the benchmark results, the worst case occurs for the values $\omega_g = 37{\cdot}3$, $\zeta_g = 0{\cdot}3$. For the other five performance indexes, the normalization values x_{3o}, $\dot{x}_{3o}$ and $\ddot{x}_{a3o}$ are the maximum relative displacement and velocity and the maximum absolute acceleration, respectively, recorded on the uncontrolled structural system.

4. DESIGNING THE FUZZY CONTROLLER

The controller was initially designed using three membership functions for each input variable and five of them for the output signal (case A). In a second phase (case B) five membership functions were also introduced for the input variables. The input/output subsets are: NL = negative large values, NE = negative values, ZE = zero value, PO = positive values, PL = large positive values. Only NE, ZE and PO are used when three membership functions are adopted for the input variables (case A). The knowledge base requires

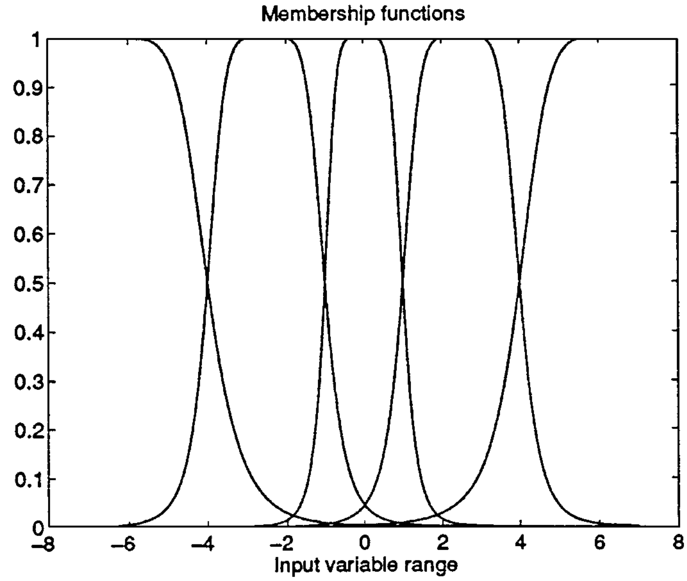

Figure 1. Input variables membership functions for the second controller design

Table I. Fuzzy rules for the first controller design

		for NE	$\dot{x}_2$ ZE	PO
and $\dot{x}_3$	NE	ZE	NE	NL
	ZE	PO	ZE	NE
	PO	PL	PO	ZE

Note: The matrix assigns a membership function to the control signal

a full understanding of the system dynamics. For this purpose a sensitivity analysis of the structure was conducted in order to emphasize the basic features of its structural response.

The benchmarck spirit suggested the authors to select, as primary target, that the fuzzy controller were able to reproduce the control u_R obtained by the use of the LQG controller of reference.

Using the signal u_R, as target, a linear regression was used to find the dependence of the control force on the measured quantities and/or their consequent velocities. The best results were achieved making use of the three storey velocities $\dot{x}_1$, $\dot{x}_2$, $\dot{x}_3$. Indeed, the weight of $\dot{x}_1$ is small in comparison with the weight of $\dot{x}_2$ and $\dot{x}_3$. The controller was therefore designed as driven by the second and third storey velocities. The ratio of the coefficients of the two velocities was approximately 8, that means that the main contribution to the control signal is due to the third storey velocity.

For case A (with three input membership functions) the normalization coefficients $\alpha_2 = 6{\cdot}24$ and $\alpha_3 = 0{\cdot}78$ were introduced for the second and third velocity, respectively. For case B (with five input membership functions) the normalization coefficients are $\alpha_2 = 2{\cdot}678$ and $\alpha_3 = 0{\cdot}334$. The resulting normalized value is used to enter the membership function of Figure 1 for case B; in a similar plot with three membership functions for case A.

Using the expertise previously collected,[2] the input and output membership functions were modified to improve the controller design by a trial 'wait and see' scheme. The adopted inference rules are summarized in

Table II. Fuzzy rules for the second controller design

		for $\dot{x}_2$ NL	NE	ZE	PO	PL
and $\dot{x}_3$	NL	ZE	NE	NL	NE	NL
	NE	NE	ZE	NE	NE	NE
	ZE	PL	PO	ZE	NE	NL
	PO	PO	PO	PO	ZE	PO
	PL	PL	PO	PL	PO	ZE

Note: The matrix assigns a membership function to the control signal

Table III. Specification of the fuzzy logic adopted in the numerical example for the two fuzzy controllers

		fuzzy subsets
Input variables	Velocity $\dot{x}_3$	3 or 5
	Velocity $\dot{x}_2$	3 or 5
Output variables	Control signal	5
Fuzzy inference	Larsen's Rule	
Defuzzification	COG	

Note: COG means center of gravity

Table I for case A and in Table II for case B. Finally, the control signal u is passed through a zero-order (case B) or first-order (case A) hold (ZOH/FOH) to stabilize the signal. This is due to the fact that the assigned SIMULINK program uses an integration step d$t = 0{\cdot}0001$ s while the control signal is computed every 0·001 s. Using the ZOH device, a constant behaviour between two calculated values is assumed, whereas a linear behaviour is given with a FOH device. The authors decided to use the ZOH and FOH devices, even though it introduces a delay in the control signal, because, without such a device, the fuzzy signal presents spikes that generate an excessive acceleration of the AMD.

5. NUMERICAL RESULTS

The structural problem is the three-floor building (defined in Reference 9) controlled by an active mass driver (AMD) located on the top floor. The governing relations are equations (1) and (2). The fuzzy logic adopted is specified in Table III.

The fuzzy controller is implemented into the SIMULINK[11] code provided in the benchmark problem by two MATLAB[12] functions. In the first function the input/output membership functions and the fuzzy rules are stored. The second function contains the procedure for evaluating the control signal: the normalization factors of the input variables are first defined; the fuzzification of the input variables and their combination by the Larsen rule is conducted and finally the defuzzification of the control signal (with the calculation of its crisp value) is provided. This second MATLAB function is inserted into the SIMULINK code replacing the LQG controller sample provided in the original program. The controller action time step $T_{\mathrm{samp}} = 0{\cdot}001$ s is also defined in the function and it is ten times bigger than the selected integration step d$t = 0{\cdot}0001$ s. Finally, the control signal is considered to have a constant or linear value from one controller output to the next one and this is done by the ZOH or FOH block available in SIMULINK.

Table IV. Evaluation criteria for a simulated earthquake using the nominal values $\omega_g = 37{\cdot}3$ rad/s, $\zeta_g = 0{\cdot}3$, and $T_f = 300$ s (no maximization over (ω_g, ζ_g) was done) and comparison between the two designed fuzzy controllers and the sample LQG

	Case A	Case B	LQG Controller
J_1	0·3508	0·3232	0·283
J_2	0·5524	0·5087	0·440
J_3	0·4093	0·4894	0·510
J_4	0·3600	0·4137	0·513
J_5	0·5667	0·5891	0·628

Table V. Evaluation criteria for the El-Centro and Hachinohe records and Comparison between the two design fuzzy controller and the suggested LQG controllers

	Case A		Case B		LQG Controller
	El-Centro	Hachinohe	El-Centro	Hachinohe	
J_1	0·4610	0·4959	0·4158	0·4748	0·456
J_2	0·8423	0·8972	0·8145	0·8666	0·681
J_3	0·5531	0·4632	0·5428	0·6249	0·669
J_4	0·5216	0·5086	0·5847	0·6474	0·771
J_5	1·1134	1·3866	1·1000	1·2994	1·280

Table VI. Active Mass Driver response for case A fuzzy controller

	Simulation	El-Centro	Hachinohe
σ_u(V)	0·1300	0·1562	0·0814
$\sigma_{\ddot{x}_{am}}$(g)	1·0145	1·2024	0·7813
σ_{x_m}(cm)	0·5362	0·6271	0·3200
max $\lvert u\rvert$ (V)	0·7477	0·5748	0·2075
max $\lvert\ddot{x}_{am}\rvert$ (g)	5·9272	5·6229	3·5774
max $\lvert x_m\rvert$ (cm)	2·6389	1·8641	0·7689

Note: The units are V, cm, and $g = 981$ cm s^{-2}

Table VII. Active Mass Driver response for case B fuzzy controller

	Simulation	El-Centro	Hachinohe
σ_u(V)	0·1580	0·1764	0·0953
$\sigma_{\ddot{x}_{am}}$(g)	1·0545	1·1600	0·8260
σ_{x_m}(cm)	0·6411	0·7192	0·3758
max $\lvert u\rvert$ (V)	0·7814	0·5506	0·2824
max $\lvert\ddot{x}_{am}\rvert$ (g)	5·2670	5·5550	3·3525
max $\lvert x_m\rvert$ (cm)	2·7661	1·8291	1·0374

Note: The units are V, cm, and $g = 981$ cm s^{-2}

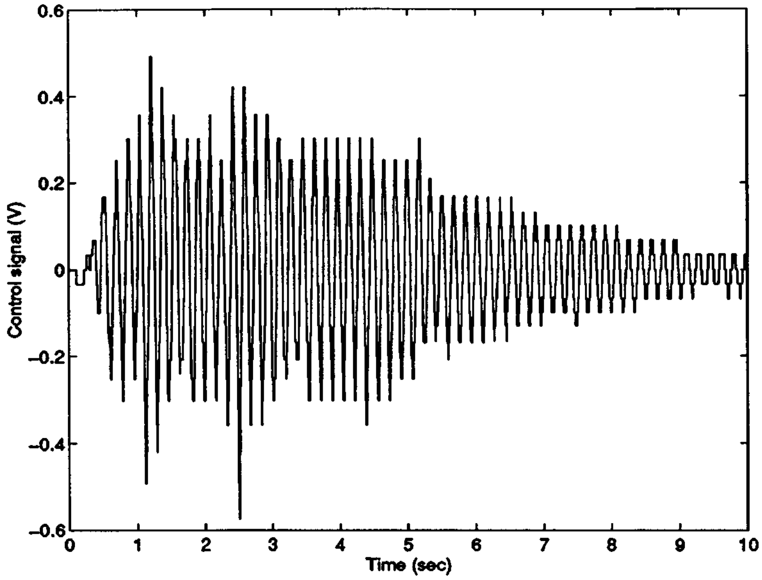

Figure 2. Control signal obtained during the simulation with the structure subjected to the El-Centro record

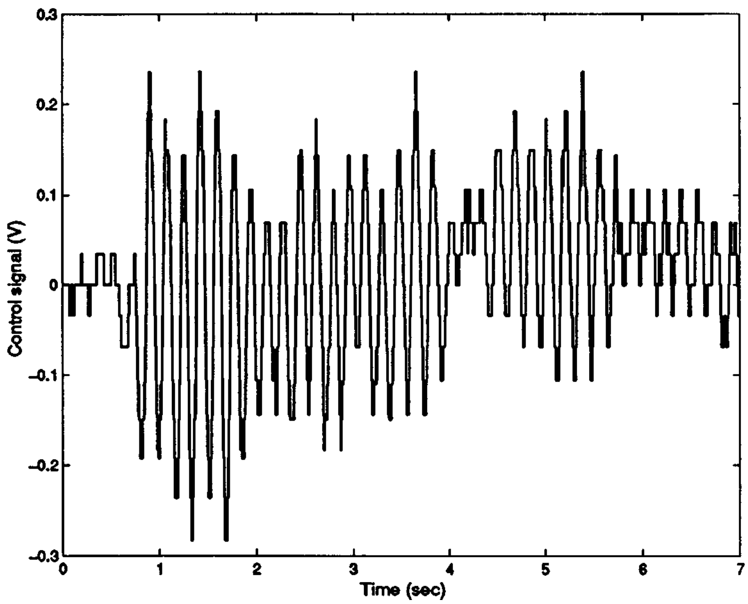

Figure 3. Control signal obtained during the simulation with the structure subjected to the Hachinohe record

5.1. Controller performance

The performance of the fuzzy controller is checked following the evaluation criteria required in the benchmark specification. The required performance indexes are shown in Table IV–VII ($\max|u(t)| \leqslant 3$ V $\max|x_m(t)| \leqslant 9$ cm, $\max|x_m| \leqslant 6$ g). All the limitation regarding the AMD behaviour are respected with the fuzzy controller.

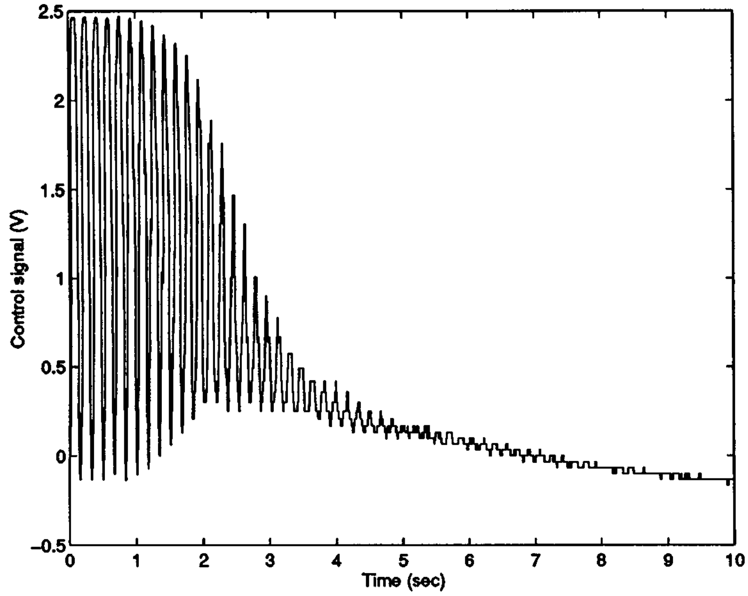

Figure 4. Fuzzy controller stability test for the initial condition $x_0(5) = 1$

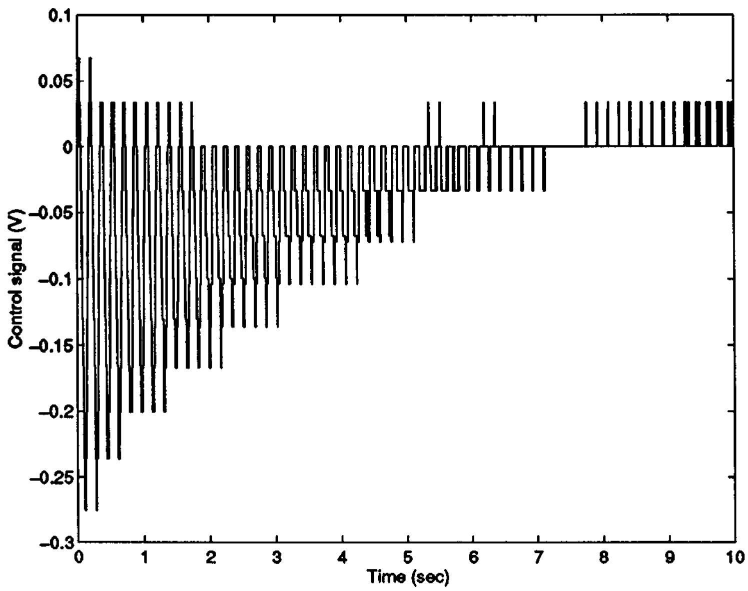

Figure 5. Fuzzy controller stability test for the initial condition $x_0(23) = 1$

The control signal u used during the simulation subjected to the El-Centro Earthquake is represented in Figure 2, while the one associated with the Hachinohe earthquake is shown in the Figure 3.

5.2. *Stability of the fuzzy controller*

Few methods are available that guarantee or check stability of fuzzy controllers. Validation is performed with simulations and tests.[13]

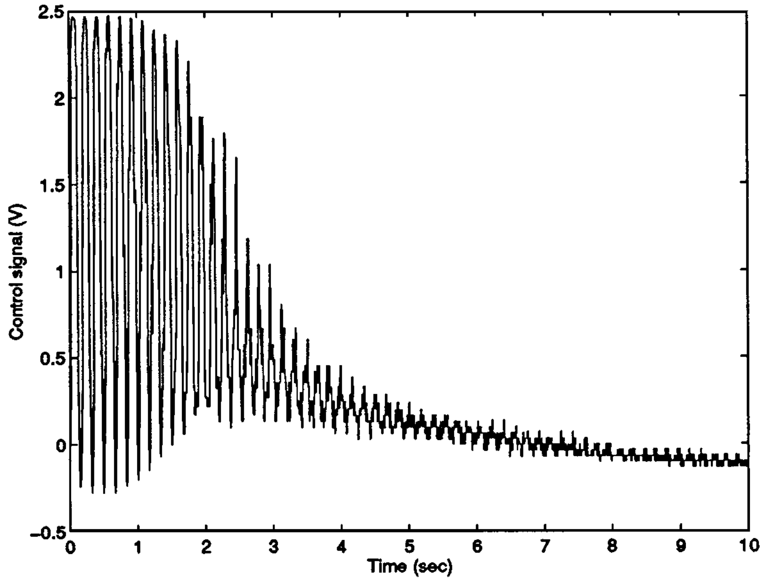

Figure 6. Fuzzy controller stability test for the initial conditions $x_0(5) = 1$ and $x_0(23) = 1$

The control stability must be checked as the ability of the controller system to return at rest from initial conditions that were caused by the external disturbance. In practice ones runs the dynamic simulation, selects the state variables that seems to show the worse response and then runs the controlled system using the worse values of the selected state variables. The test consists of checking the ability of the controller to reduce the response and to drive the system to drive the system to the rest position after the initial transient phase.

The stability tests are performed considering the system with particular initial conditions on the state vector x and checking the ability of the controller to reach the equilibrium after the initial transient phase as shown in the Figures 4–6.

Non zero initial conditions $x_o(i)$ are assigned to the components of the state vector $\mathbf{x}$ that maximize the controller action, i.e. $x(5)$ and $x(23)$.

6. CONCLUSIONS

The results presented in this work show how the use of a fuzzy control approach can represent a possible way to control the response of a structural system controlled by an active mass driver. The advantage of the proposed approach is essentially the limited number of measured structural responses (the two-storey velocities) used to implement the control rules and its intrinsic robustness. An extension to incorporate geometric and material non-linearities does not require any modification in the control scheme.

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REFERENCES

1. B. M. Ayyub and M. H. M. Hassan, 'Control of construction activities: III. A fuzzy based controller', *IEEE Trans. Systems Man Cybernet*. **20**(2), 404–435 (1990).

2. L. Faravelli and T. Yao, 'Use of adaptive networks in fuzzy control of civil structures', *Microcomput. Civil Eng.* **11**, 67–76 (1996).
3. R. S. Subramanian, A. M. Reinhorn, S. Nagacajaiah and M. A. Ryley, 'Hybrid control of structures using fuzzy logic', *Microcomput. Civil Eng.* **11**(11), 1–18 (1996).
4. F. Casciati, L. Faravelli and T. Yao, 'The effects of nonlinearities upon fuzzy structural control', *Nonlinear Dyn.* **11**, 171–187 (1996).
5. F. Casciati and L. Faravelli, 'Fuzzy control of nonlinear systems in the presence of noise', *Des. Eng. Tech. Conf.* Vol. 3, Part A, ASME, De- Vol. 84-1, 1995.
6. L. Faravelli and T. Yao, Self-learning control of civil structures, *Proc. ISUMA-NAFIPS'95* College Park, Maryland, USA, 17–20 September 1995.
7. F. Casciati and F. Giorgi, 'Fuzzy controller implementation', *2nd Int. Workshop on Structural Control*, IASC, Hong Kong, 18–21 December 1996.
8. T. Takagi and M. Sugeno, 'Derivation of fuzzy control rules from human operator's control actions', *Proc. IFAC symp. on Fuzzy Information, Knowledge Representation and Decision Analysis*, 1983, pp. 55–60.
9. B. F. Spencer Jr, S. J. Dyke, and H. S. Deoskar, 'Benchmark problems in structural control: Part I — Active mass driver system', *Earthquake Eng. Struct. Dyn.* **27**, 1127–1139 (1998).
10. L.-X. Wang, Adaptive Fuzzy Systems and Control, Prentice-Hall, Englewood Cliffs, NJ, 1995.
11. SIMULINK, The Math Works, Inc. Natick, Massachusetts, 1994.
12. MATLAB, The Math Works, Inc. Natick Massachusetts (1994).
13. F. Casciati, 'Checking the stability of a fuzzy controller for nonlinear structures', *Microcomput. Civil Eng.* **12**, 205–215 (1997).
14. F. Casciati and L. Faravelli, *Fragility Analysis of Complex Structural Systems*, Research Press, Taunton, England, 1991.
15. S. J. Dyke, B. F. Spencer Jr., P. Quast, M. K. Sain, D. C. Kaspari Jr and T. T. Soong, 'Acceleration feedback control of MDOF structures', *J. Eng. Mech. ASCE*, in press.

